GS02 1052 – Imaging Science

Course description: This course provides a concise and coherent review of some commonly-encountered topics in applied mathematics, with a particular emphasis on their applications and relevance to imaging science. The course covers and is equally divided into two major sections: (i) image analysis methods and (ii) Fourier and wavelet transforms. Our image analysis approach provides a formalism for image registration, image reconstruction, image segmentation, and machine learning.

Learning objectives:

- Develop a mathematical formalism for approaching some commonly encountered problems in medical physics
- Interpret multiple distance measures for identifying accurate solutions
- Apply the mechanics of a neural network for image segmentation
- Understand common computational techniques for solving optimization problems inherent to image registration and image reconstruction
- Understand the mathematical formulation, properties, and limitations of Fourier transforms (continuous and discrete) and wavelet transforms and how they relate to some medical physics problems

Semester Begins: Monday, Aug. 30, 2021

Time and location: 11:00am-12:00pm (Tuesday, Friday) via zoom

Syllabus:

Class	Instructor	Topic
1 (8/31)	Fuentes	1. Preliminaries - Algorithm Complexity, BLAS
2 (9/3)	Fuentes	2. Vector and metric spaces
3 (9/7)	Fuentes	3. Mutual Information
4 (9/10)	Fuentes	4. Entropy
5 (9/14)	Fuentes	5. Linear Independence, Equivalence of Norms
6 (9/17)	Fuentes	6. Linear Operators, Convolution
7 (9/21)	Fuentes	7. Image Segmentation
8 (9/24)	Fuentes	8. Analysis of Neural Network Structure
9 (9/28)	Fuentes	9. Rank and Nullity, Bounded Operator, Stability of Linear Systems
10 (10/1)	Fuentes	10. Inner Product, Orthogonality, Eigen-formulation
11 (10/5)	Fuentes	11. Optimization Characterization of Solution
12 (10/8)	Fuentes	12. Line search, Newton-CG Trust-Region Methods
13 (10/12)	Fuentes	13. Least Square, QR Decomposition

14 (10/15)	Fuentes	14. L1 minimization
15 (10/19)	Fuentes	15. exam on optimization
16 (10/22)	Mirkovic	1. Where Nature does or calls for FT, definition of FT, notations, symmetry and FT, interpretation of FT
17 (10/26)	Mirkovic	2. Some useful functions and calculating their FT
18 (10/29)	Mirkovic	3. Convolution, correlations, the central limit theorem
19 (11/2)	Mirkovic	4. LTI systems, more on convolution, FT as a linear system, what's special about exp(-i2πxy)?
20 (11/5)	Mirkovic	5. Convolution theorem and other Fourier theorems
21 (11/9)	Mirkovic	6. The δ-function, distributions/generalized functions, re-defining FT
22 (11/12)	Mirkovic	7. Discrete Fourier Transform, cyclic convolution
23 (11/16)	Mirkovic	8. The Fast Fourier Transform algorithm
24 (11/19)	Mirkovic	9. The Sampling Theorem, ghost function, and Nyquist frequency
25 (11/23)	Mirkovic	10. Drawbacks of FT, wavelets, and continuous wavelet transforms, wavelet transform as a cross-correlation and constant-Q filtering
26 (11/30)	Mirkovic	11. Discrete wavelet transforms and orthogonal wavelet decomposition, multi-resolution analysis
27 (12/3)	Mirkovic	12. more on MRA, scaling function, digital filtering
28 (12/7)	Mirkovic	13. Haar wavelets, filters, filter banks, multistage filter banks, and perfect reconstruction
29 (12/10)	Mirkovic	14. Medical physics applications of Fourier and wavelet transforms
30 (TBD)	Mirkovic	15. Exam on transforms

Grading:

Homework: 40%; Exams (in-class): 60%

Major references:

[1] Jorge Nocedal and Stephen J. Wright. *Numerical optimization*, 2nd edition, Springer Verlag, 1999.

[2] Ronald Bracewell, *The Fourier transform and its applications*, 3rd edition, McGraw Hill, 2000.

Additional references:

[1] Michael Greeberg, *Foundations of Applied Mathematics*, Prentice Hall, 1978

[2] E. Kreyszig, *Introductory functional analysis with applications*, volume 21. Wiley, 1989

Holidays: Labor Day 9/6/2021; Thanksgiving 11/25-26/2021